Use of the Maximum Likelihood Method for the Statistical Evaluation of Fatigue Tests

Moving on from Laboratory Test Data Storage to a Corporate Materials Knowledge Management

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Abstract:
Applying the maximum likelihood method fatigue tests can be statistically evaluated. In the past, this method has proven to be an excellent tool to also evaluate those data not perfectly matching the requirements of DIN 50100. The aim is to implement the maximum likelihood method in a knowledge system for materials using the example of the evaluation of S-N curves of ductile cast iron. The solution approach covers the entire process from the raw data from the testing machine to the evaluation and the provision of comparable knowledge and characteristic values. The characteristic values are saved in a way ready to be provided to other (FEM-) Simulation tools. In addition to the maximum likelihood method, the characteristic values will also be evaluated according to other methods and the results of the different methods can be compared with each other.

Keywords: Maximum likelihood method, material database, test evaluation, knowledge management, test documentation

1 Introduction

The importance of material characterization as a whole is increasing: Increasingly powerful CAD/CAE and simulation tools demand even more specific and reliable material data with more detailed consideration of the process history. The "digital twin" is the overarching goal of today's holistic product development activities. Materials testing for the description and verification of central properties of design and functional materials is of central importance here, alongside design, materials, process and manufacturing simulation. Even though the introduction of electronic data acquisition in materials testing has been state of the art for over 30 years, inconsistent data models, decentralized storage and media discontinuities often lead to problems when implementing a "fully digital" process chain. In many cases, reports and certificates are only generated locally using office tools and stored "digitally". The reports are often created in the context of the project and cannot be linked to other activities in the company. Expensive raw data is usually completely lost or lives out its existence in local, unconnected data silos.

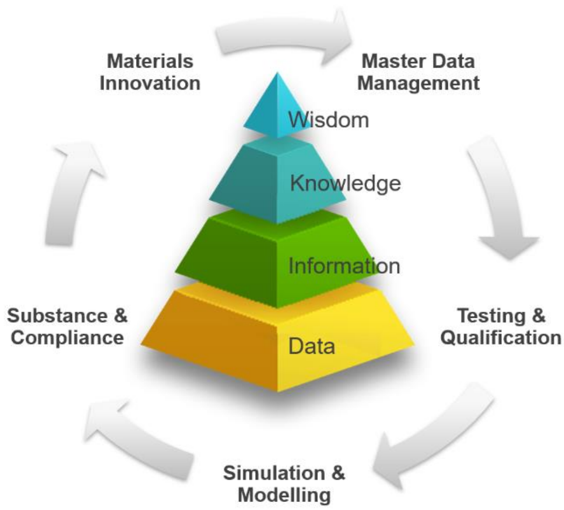

Fig. 1: Knowledge Pyramid – from "Data Lakes" to Corporate Material Wisdom [1]

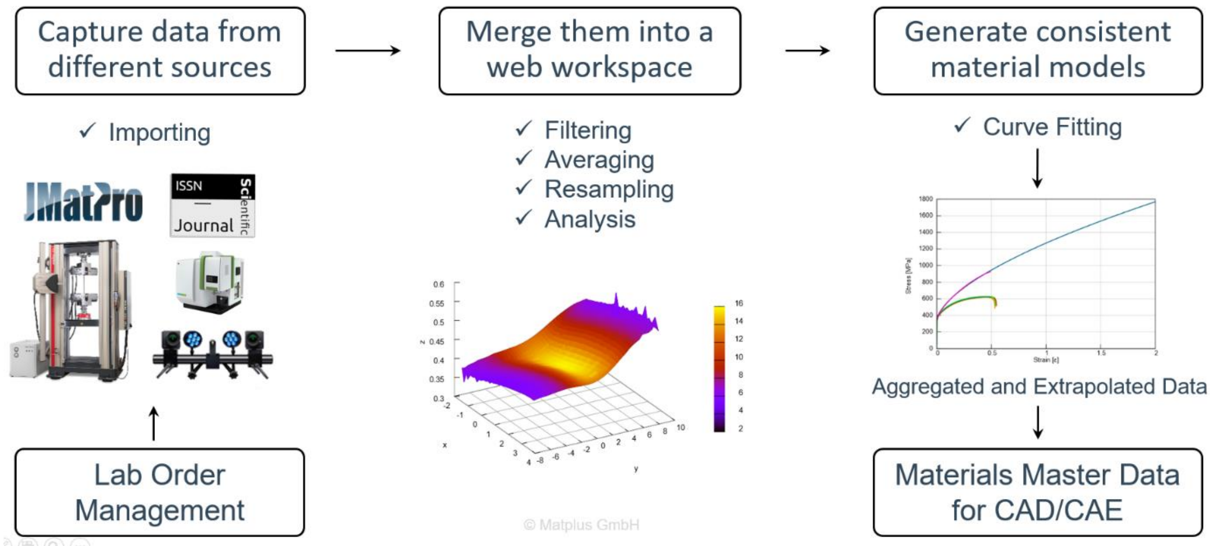

Fig. 2: Schematic workflow for importing materials data of different types; analysis and modelling in the EDA® system [2, 3]

Digitization describes the pure analog-to-digital conversion of existing data and documents with the sole advantage of full-text search compared to the paper form. Beyond that, our goal is digitalization: A transformation into digital business processes for materials engineering. This includes the continuous creation and use of a knowledge base and makes "materials analytics" possible: Acquiring insights and knowledge from order- and project-wide information, e.g. by combining data from materials testing with data from the process chains for the production of components, materials and samples. The resulting knowledge base allows for a comprehensive, person-independent core competence of a materials processing company that sustainably secures and values creation through innovation.

Fig. 1 shows the structure of such a core competence in the form of a "knowledge pyramid". Identified data are the indispensable foundation - it must be as complete and comprehensive as possible for all processors. But real information can only be obtained through analysis and linking, e.g. of material and process data. This information must be condensed and processed as a report for company-wide decision-making - ideally, however, all the data from the reports can also be traced back "to the source" at any time and processed further. The more solid these building blocks are, the more reliable innovations will emerge from a deep understanding of the processes in and around the own product.

2 Materials Information Management

Digitized testing machines record much more data than the certificate-relevant characteristic values. In addition to the various metadata/header data of a test a multitude of time-dependent data are recorded via different channels, e.g. force, time, displacement, temperature, stress and strain. In the context of the increasingly important use of optical extensometers or DIC (Digital Image Correlation), a combination of these signals with the signals from the testing machine seems useful.

In addition, calculated material data via programs such as JMatPro®, Matcalc® or ThermoCalc® which can be combined with test data and verified on the basis of these data or complement them in a meaningful way. Fig. 2 shows the workflow of such a procedure for plasticity of materials in EDA®, which is the materials information system created by Matplus. The aggregated data can be processed and analyzed by means of EDA® and either, as shown here, combined into data models for a CAD/CAE environment, or can be evaluated by development teams in a variety of ways.

It is good practice to store the generated data sustainably in archivable formats rather than using testing machine-specific binary formats. JSON (Javascript Object Notation) is a universal technology that is particularly suitable for materials testing data. Even sophisticated parametric data fields with high-resolution, time-dependent data can be efficiently stored as structured text documents in a human- and machine-readable format.

EDA® provides a variety of import filters or converters that transfer data from different formats, e.g. also SEP1240 [4], into the internal JSON representation. These importers can be flexibly extended and adapted to support the testing machines of different laboratories and manufacturers as well as foreign formats. The JSON schema is freely customizable, so that special tests with special channels can also be integrated.

The data is stored in the NoSQL database MongoDB, which is characterized by very good scalability, so that even large data volumes in the multi-GB range can be processed efficiently. In the case of extensive material qualification projects, separate databases can also be defined in each case.

Test results are assigned several different tags/identifiers in the system, so that they can be displayed and evaluated both on an order-related and an overall basis. For this purpose, freely definable input masks can be used to define the orders, test series and individual tests. Furthermore, such metadata as well as the test data itself can also be automatically transferred from external systems via a REST interface.

To evaluate the test data with the intention to produce data needed by other simulation tools, different methods need to be applied. In the case of S-N fatigue tests this can be the maximum likelihood method [5] to define the parameters of the S-N curve. Other methods are given in DIN 50100 [6], as there are the Probit, the delimination or the staircase method.

3 Evaluation Applying Maximum Likelihood Method

When estimating S-N curves based on fatigue tests, the evaluation of run outs plays a significant role. Run outs are specimens that have not failed after a specific number of cycles at which the test was stopped. Therefore, for these specimens, there is no information on the actual number of cycles to failure. This type of data is called censored data and cannot be evaluated by classical regression without systematical bias. However, the information on the endured number of cycles can be statistically considered and the systematical bias can be avoided, e.g., by the maximum likelihood estimation (MLE) [5]. In general, MLE can be used to estimate the parameters p of a model $y = g(p|x)$ based on the samples x and the assumption of a probability distribution by maximization of the likelihood function

$$L(x|p) = \prod_{i=1}^{N} P\,(x_i|p), \tag{1}$$

where N is the number of samples x. In simple applications, the maximum of the likelihood function is obtained by differentiation and an analytical solution. For more advanced applications, the maximum is obtained by numerical optimization. In the second case, the likelihood function (1) is evaluated for different parameters till the maximum is found, i.e., the optimization has converged.
As shown in [5], the probability density function (PDF) of the logarithmic normal distribution

$$P_{failure}(x|p) = f(x|p) = \frac{1}{\sqrt{2\pi\hat{s}^2}} e^{-\frac{\hat{\epsilon}^2}{2\hat{s}^2}}, \tag{2}$$

is suitable for the estimation of S-N curves, where $\hat{\epsilon}$ is the logarithmic difference between the estimated and measured model output y and s is the estimated standard deviation as shown in (3) and (4). In addition, equation (3) can be interpreted as cost function. However, it must be mentioned that the PDF outputs a relative probability as a result. The absolute probability of occurrence for a single experimental result can only be calculated by integrating the PDF within narrow limits. For the application within the MLE this can be neglected [5].

$$\hat{\epsilon} = log_{10}y - log_{10}\hat{y} \tag{3}$$

$$\hat{s} = \tfrac{1}{N}\sum\hat{\epsilon}^2. \tag{4}$$

The probability $P_{failure}$ of failure of a specimen or component for a given number of cycles N and a given stress level σ_a can be calculated directly using the equations (2), (3) and (4). The probability P_{runout} of a run out is calculated by the cumulative density function (CDF), i.e., the primitive of the probability density function (2) and can be interpreted as the probability that a specimen or component survives the endured number of cycles. Since the CDF gives the probability for a specimen to fail inside a specific number of cycles, the probability of survival is given by

$$P_{runout}(x|p) = 1 - F(x|p). \tag{5}$$

There are different ways to apply the concept of MLE for S-N curves, e.g., the application for different formulations of the S-N curve after the knee point. In the framework of PyLife [7], a slope after the knee point is not considered. The S-N curve is defined by

$$\sigma_a = \sigma_{a,k}\left(\frac{N}{N_k}\right)^{-\frac{1}{k}}$$

and (6)

$$N = N_k\left(\frac{\sigma_a}{\sigma_{a,k}}\right)^{-k}$$

respectively. In maximum likelihood full estimation with PyLife, specimens which endured a higher stress than the stress at the knee point $\sigma_{a,k}$ are estimated in lifetime direction. Therefore, equation (6) is rearranged such as

$$y = N_k = N\left(\frac{\sigma_a}{\hat{\sigma}_{a,k}}\right)^{\hat{k}} \tag{7}$$

and the likelihood is calculated according to equations (1), (2) and (5). Specimens which endured a lower stress than the stress at the knee point $\sigma_{a,k}$ are evaluated in stress direction by the CDF, regardless of whether the specimen has failed or not. As a result, $1 - F$ is used for run outs and F is used for specimens that have failed normally. In addition, the cost function is changed to

$$\hat{\epsilon} = log_{10}\frac{y}{\hat{y}}. \tag{8}$$

4 Experimental

In this particular case, an importer for raw data of S-N fatigue tests has been developed. Subsequent evaluation of these data applying the maximum likelihood method is performed to create a master material data sheet with the characteristic data. These can be exported to other simulation tools.

The importer is intended for the data originally generated from fatigue testing machines (e.g. resonance or servo-hydraulic test rigs, etc.) to examine the fatigue strength of the specimen. It is not an obligation that tests should have to be done only by above mentioned machine to evaluate the structural durability in EDA®. EDA® is quite versatile regarding input data format. The user can not only upload individual data, but also a set of compressed data. Each test is saved as a JSON file that can be used for post-processing. For this purpose, an add-on plugin "Woehler Curve" has been created, which uses the required data stored in "Test Summary" from the individual test data based on the predefined specimen failure criteria.

4.1 Fatigue test analysis

It has been discovered in the mid-20th century that most material failures are due to fatigue [8]. As a result, today's product development relies heavily on numerical methods used in fatigue simulations, rather than relying totally on destructive or non-destructive inspection tests and using those tests results to predict the lifetime of components under cyclic loading. Finite Element Method (FEM) is a well know numerical technique used for solid body simulation. Practical experiments are equally important as numerical simulation methods, because simulation results can hardly be validated without practical testing. Since mid-1920s, numerous research efforts have been dedicated to figure out the unpredictable fatigue failure behavior, yet this phenomenon is still being explored.

It is known that repetitive cyclic mechanical loading reduces the life of a specimen compared to the same specimen subjected to the equivalent maximum force, but in the static case. Fatigue failure is a result of (highly localized) plasticity, which is a result of geometrical non-linearity such as sudden shape change, which creates a bottle neck at a certain point. It is also called "stress concentration". At micro scale geometrical non-linearity could be defined by morphology of microstructure. A material non-linearity also plays role in the fatigue failure. Within elastic regime an elasto-plastic material model could be considered linear, but after yield criteria (von Mises or Tresca) a material becomes nonlinear and has to be defined using suitable nonlinear material models. Therefore, for cyclic loading conditions, regardless of elastic or plastic response of material, prediction of structural durability is crucial to avoid catastrophic failures.

The crack evaluates from nucleation to macro scale and lifetime of specimen ends. Fatigue analysis is possible using stress-life (SN) relation and strain-life (ϵN) relation. A stress-life relation is mostly used in

a situation where plastic strain occurs only at the tips of fatigue crack and it is useful at medium (MCF), high (HCF) and very high cycle fatigue (VHCF). In case of low cycle fatigue (LCF), a strain-life relation is preferred.
An LCF, i.e. ϵN-approach, considers elastic as well as plastic strain of the material whereas magnitude of plastic strain reduces relatively faster than elastic strain with respect to number of cycles therefore elastic strain becomes dominant after knee point. As it can be seen in an example of Manson–Coffin–Basquin curve given by A. Nieslony *et. al* [9], elastic strain ($\epsilon_{a,e}$) and plastic strain ($\epsilon_{a,p}$) amplitudes show linear behavior.

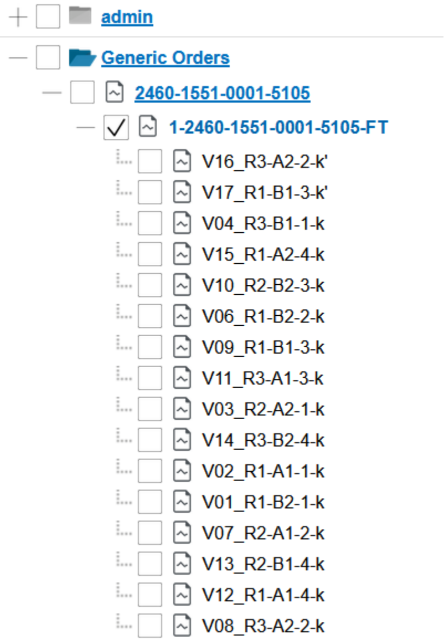

Fig. 3: Tree structure of imported test data

In the transition of HCF (High Cycle Fatigue) to VHCF (Very High Cycle Fatigue), the slope of the S-N curve changes considerably. A VHFC region is one in which induced stress is below permissible stress limit of the material and material generally has a small elastic strain. Therefore, crack evolution occasionally happens. In case of load-controlled fatigue testing, an evaluation of cyclic tests is done at constant load amplitude. Even though with same external load and identical boundary condition, every test result may vary which makes statistically based scatter bands with a distinct survival probability necessary. Therefore, to generate a continuous function of stress and lifetime using discrete points one can use some statistical estimations of constants if they are unknown. Now, along with other features, EDA® also provides a possibility to find the constants of the Woehler curve. For this purpose, the "Scientific Python" based infrastructure of the EDA® platform was supplemented by the free module "pyLife" [7].

After uploading the data using the provided importer/uploader, the required constants of the Woehler curve can be identified in a few steps using a plugin. EDA stores the test data in a tripartite LIMS (Laboratory Information Management System) -type

SN-Curve

Full Menu · Toggle Headings · Data · Attributes · Graph Data · Stat Data · Load Table Filter · Evaluation

Cycles to Failure	Load Amplitude [N]	Stress Amplitude [MPa]	Test Name	Comments
1.000007e+7	5635.6654	206.586	V08_R3-A2-2-k	Run Out
956950	7075.0854	259.3506	V12_R1-A1-4-k	
188675	7346.4419	269.2977	V13_R2-B1-4-k	
560516	6785.6706	248.7416	V07_R2-A1-2-k	
721130	6220.0372	228.0072	V01_R1-B2-1-k	
1491080	6219.9076	228.0025	V02_R1-A1-1-k	
181349	7058.1978	258.7316	V14_R3-B2-4-k	
814169	5898.072	216.205	V03_R2-A2-1-k	
30534	8454.4678	309.9145	V11_R3-A1-3-k	
1.000006e+7	5776.7763	211.7587	V09_R1-B1-3-k	Run Out
255119	6785.7313	248.7438	V06_R1-B2-2-k	

Fig. 4: Test summary based on predefined failure criteria

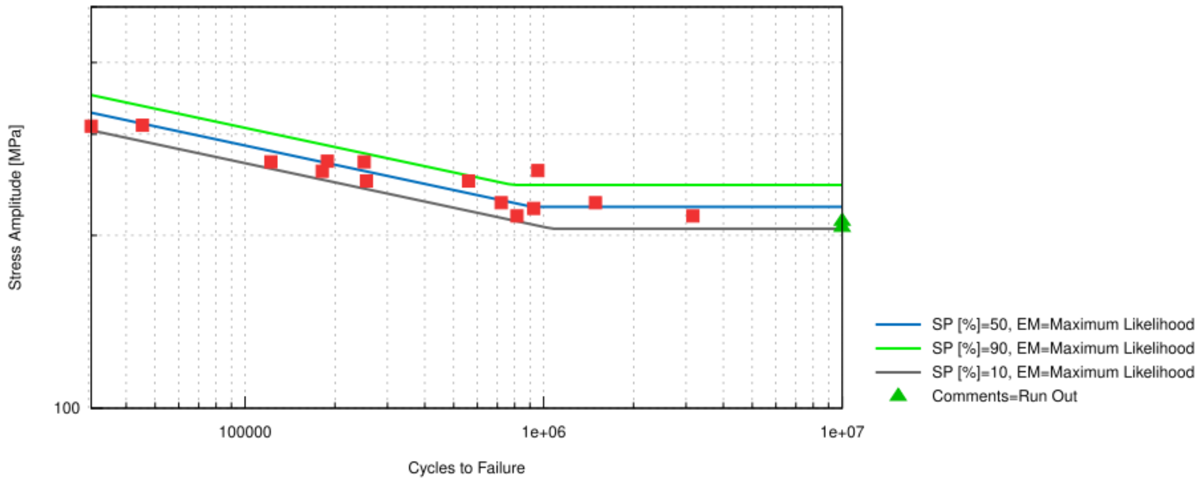

Fig. 6: S-N curve in EDA evaluated with maximum likelihood method

tree structure, starting from a test order via test series to individual tests.

For every test summary (**Error! Reference source not found.**), a number of fatigue test data (lifetime and load amplitude) is taken from individual experimental test based on predefined failure criteria of specimen.

The "Woehler" plugin assists in determining the required constants for the Woehler curve:

- Selection of the statistical method for constant search (inspired by pyLife [7]) or

- Input of already known predefined constants like e.g., the stress amplitude at the knee point as shown in Figure 5.

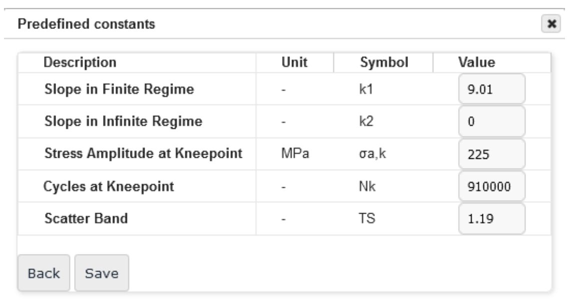

Fig. 5: Required predefined constants for Woehler curve representation

Based on the selected evaluation type, the methods and the constants in section tag *Cyclic characteristics, Woehler* and in *Curve* tab EDA® will show the respective representation (**Error! Reference source not found.**). The key parameters to represent the S-N curve shown in Figure**Error! Reference source not found.** are calculated using maximum likelihood method. Also, the same parameters have been calculated using "elementary" and "maximum likelihood infinite method" which all are available in pyLife [5].

The elementary method is mainly used to generate initial parameters for maximum likelihood estimation. It is based on the evaluation of the S-N data in a normal probability plot and on regression, as shown in [10] using the pearl chain method. The maximum likelihood infinite method is a combination of the elementary method and the maximum likelihood method. In this method, the results from the elementary method are used, but the stress amplitude at the knee point ($\sigma_{a,k}$) and the scatter in stress direction (T_S) are determined using the maximum likelihood estimation. The knee point (N_k) is not directly determined by maximum likelihood estimation, it is calculated based on the stress at the knee point $\sigma_{a,k}$ and the slope in finite regime (k_1).

The difference between the key parameters of these three known methods are shown in below Table**Error! Reference source not found.**:

Table 1: Comparison of the results of different methods

Key parameters	Elementary	Max. likelihood	Max. likelihood infinite
$\sigma_{a,k}$ $[MPa]$	228.01	224.25	225.49
$N_k[-]$	$7.89x10^5$	$9.16x10^5$	$8.72x10^5$
T_s $[-]$	1.19	1.19	1.23

The slope in finite life regime for all methods is $k_1 = 9.01$. The red squares in Fig. 6 and Fig. 7 are experimental test results and the three lines are continuous closest possible curve fitting of the "Woehler curve" based on given input using elementary method. A blue line represents 50% probability of survival, whereas green and grey line represent 10% and 90% probability of survival (Fig. 6). The scatter lines are important to get an optical impression of the survival probability.

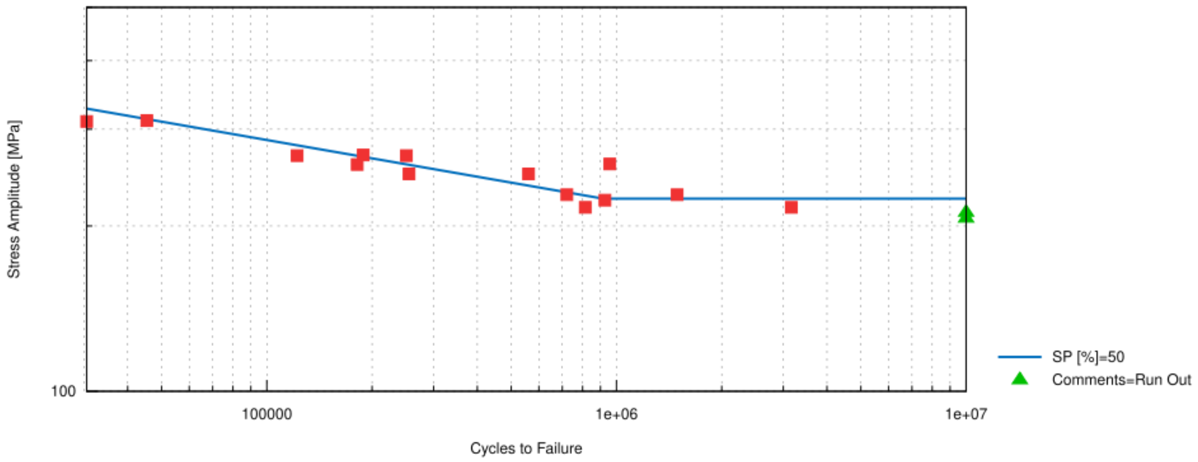

Fig. 7: S-N Curve in EDA using predefined constants

4.2 Metallography

Fatigue data results are very dependent on the microstructure, the amount of ferrite and perlite, the number, size and shape of the graphite particles as well as grain size of the material [11]. This makes the microstructure an essential part of the complete data set and therefore EDA® also offers to save these data together with the raw and evaluated data (**Error! Reference source not found.**). Obviously, the attributes to describe the microstructure can be customized in case it is planned to perform fatigue tests on a different material such lamellar cast iron, steel or aluminum.

2-MultiWind_Block*-Microstructure

Alle Menüs | Editieren | Batch Import Data | Add Image | Add PDF

	2-MultiWind_Block*-Microstructure
Order	MultiWind_Block*
Versuchsreihe	Microstructure
Werkstoff	GJS 400-18C-LT
Fraction of Phase Graphite [%]	10.86
Number of Particles	1708
Code of Nodularity [%]	51.9
Fraction of Phase Ferrite [%]	58.8
Fraction of Phase Pearlite [%]	3.4
Number of Fields (unetched)	10
Number of Fields (etched)	10
Auswertung nach Norm	EN ISO 945

It is also rendered possible to save pictures together with the data set (**Error! Reference source not found.**) with allows the direct comparison to the evaluated data of Fig. 9.

To compare the results of different tests of one material or different material groups this can be performed using the data comparison table (Optimizer). Here, characteristic data can be compared, statistically evaluated according to the standards (e.g. [12]) and plotted, also in parametric form.

4.3 Integrated reporting

All determined evaluations, data tables and image files can be integrated into reports generated by the system. The complete LaTeX library is available for this purpose. The result is professional, print-ready documents. The main advantage, however, is that all data sets contained in the report are accessible from within the report - if the report is made accessible from within the system, which is easily possible through a fine-grained customizable permission system, all data sets and evaluations can be traced back to the data source with one click, provided the reader's permission allows this. Reports thus become a true store of knowledge; data from reports can be quickly used for further analysis. Extensive literature database functions are available for the management of existing documents, such as the full-text searchability of indexed pdf files. An existing reporting system can thus be seamlessly integrated into the EDA® environment.

cases and should be used preferentially, even if only minor differences can be seen in the example shown here.

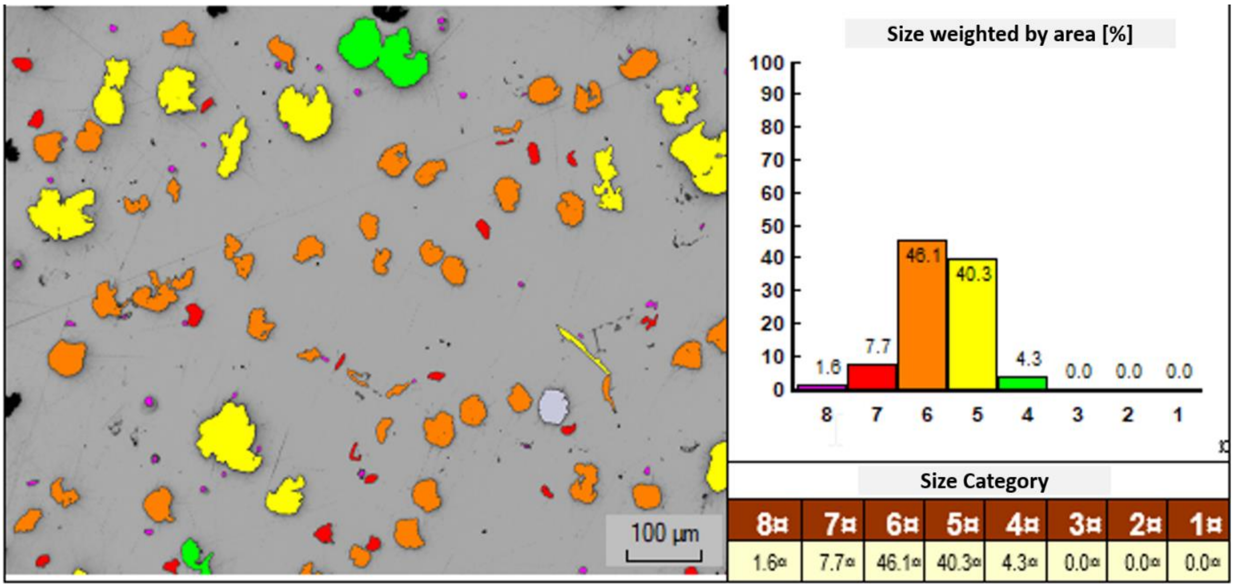

Fig. 8: Evaluation of the graphite particle size

5 Conclusions

The Matplus EDA® system enables the integration of information from materials testing into a comprehensive knowledge base and links this with materials and process data. Valuable core know-how is thus made available throughout the company with vertical and horizontal integration across project and departmental boundaries. Data and knowledge islands are dissolved and the uniform use of consistent, verified models is made possible. In addition to simplifying processes, the system ensures a uniform data structure in formats that can be read at any time without proprietary binary objects and thus the best possible sustainability.

For the field of fatigue strength, cyclic test results from testing machines can be imported. Integrated functionalities for evaluation include classical methods as well as the Maximum-Likelihood method and curve fitting. Results from different evaluations can be easily compared and visualized in the web environment. The advantage is that results from different projects and reports can be directly overlaid.
Studies [5] show that the Maximum-Likelihood method is superior to the classical methods in many

6 Acknowledgement

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